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Evaluation of Stochastic Models for Estimating the Persistence Probability of Cloud-Free Lines-of-Sight

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Stochastic models, based upon the Ornstein-Uhlenbeck (O-U) class of the simple Markov process have been used effectively to estimate the conditional probability of a variety of weather events (Gringorten, 1972). This study is directed toward extension of the analytical form of the O-U Markov model to yield estimates of the joint occurrence probability and duration of sky cover and cloud free lines of sight (CFLOS). As a first step, model estimates are made of the recurrence probability of CFLOS for specific categories of sky cover in tenths as a function of time. Then model calculations are made for the persistence probability of CFLOS and sky cover as a function of sky-cover category and time using an analytical representation of the mathematic solution of persistence probability for the O-U Markov process given by Kielson and Ross (1975). The model calculations of CLFOS recurrence and persistence are compared with the Columbia, MO, data base (Lund, 1973). Again using an analytical form of the O-U Markov model, calculations are made of the joint occurrence probability of sky-cover at two or more sites and compared with the observations from the central United States. Finally, analytic techniques are presented for calculating probability estimates of the duration of cloud-free or cloudy lines of sight from one or multiple ground sites to points in space. Trial calculations based upon climatic summaries of sky-cover are made for a selected group of sites in south western United States.

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TABLE OF CONTENTS

LIST OF TABLES

List of Tables con't.

LIST OF ILLUSTRATIONS

1.0 INTRODUCTION

Minute by minute determinations of sky-cover and cloud-free-line-of-sight (CFLOS) are being made in an experimental field program initiated in 1988 by the University of California, San Diego. Solid-state, whole-sky imagery (WSI) systems have been installed in a network configuration consisting of 6 field sites in the western United States. The primary goal is to obtain detailed observations of the joint occurrence frequency of CFLOS in time and space that are required to evaluate and extend sky-cover models. The stochastic sky-cover models are to be used for estimating the impact of clouds on ground-based systems that depend upon unobscured paths of sight to satellites in space.

A comprehensive statistical model has been developed by Boehm, et al, (1986) that specifically provides estimates of the duration of cloud free lines of sight from multiple ground sites to orbiting and geostationary satellites. The innovative method establishes the climatic probabilities through repetitive simulations of sky-cover distributions with the multidimensional Boehm Sawtooth Wave Model. The simulation model and its many component approximations are the prime candidates for test and evaluation with the new WSI data base.

Prominent among alternate approaches to the problem are models based on the Ornstein-Uhlenbeck (O-U) class of the simple Markov process. This approach has been applied successfully both in purely analytical form and by Monte Carlo simulation of probability distributions to estimate the joint occurrence and duration of a variety of weather events (Gringorten 1966, 1967, 1968, 1972). This study is directed toward extension of the analytical form of the O-U Markov model to yield estimates of the joint occurrence and persistence probability of cloud free lines of sight in time and space.

The ultimate objective of the modeling process is to determine the joint climatological probability of the duration of cloud free lines of sight to a point in space from one or more preselected ground sites, given the climatic summaries of sky-cover at each site. The solution to a number of intermediate modeling problems is required to achieve the desired objective. In Section 2, the application of existing models to estimate cloud free line of sight (CFLOS) probability as a function of sky-cover and the zenith angle of the path of sight is described. The basic O-U Markov modeling concepts are reviewed in Section 3. Estimates of the single station duration of CFLOS as a function of sky-cover using the Keilson-Ross procedure are discussed in Section 4. Estimates of the joint conditional probability of sky-cover at multiple sites is discussed in Section 5. Procedures for estimating the frequency of downtime intervals caused by cloud obscured lines of sight persisting concurrently at designated network sites are described in Section 6.

2.0 CFLOS AS A FUNCTION OF SKY COVER AND ZENITH VIEWING ANGLE

Variations in the angle of view through the atmosphere coupled with the 3-dimensional structure of cloud forms result in a systematic decrease in the average relative frequency of CFLOS with an increase in the zenith angle of the ground-based observer's path of sight. The probability of CFLOS as a function of sky-cover, cloud type and zenith viewing angle was determined empirically by Lund and Shanklin (1973). Three years of hourly summer data were used to establish the model estimates. The smoothed and adjusted probabilities were derived from whole-sky photographs with infrared film and companion observations of cloud amount by National Weather Service observers at Columbia, Missouri. The relative frequencies of CFLOS as summarized for a composite of all cloud types by Lund and Shanklin are shown in Table 1. The model estimates have been refined and upgraded by Allen and Malick

Table 1. Model estimates of the relative frequency of cloud free line of sight as a function of total sky cover and zenith viewing angle.

LUND AND SHANKLIN EMPIRICAL MODEL

ZENITH ANGLE DEGREE	SKY COVER (TENTHS)						AVE
	0	2	4	6	8	10	
0	1.00	0.92	0.81	0.70	0.48	0.08	0.665
30	0.99	0.90	0.80	0.66	0.46	0.08	0.648
50	0.99	0.88	0.76	0.62	0.42	0.07	0.623
70	0.98	0.83	0.67	0.50	0.33	0.05	0.560
80	0.97	0.76	0.55	0.39	0.24	0.03	0.490

ALLEN AND MALICK GEOMETRIC MODEL

ZENITH ANGLE DEGREE	SKY COVER (TENTHS)						AVE
	0	2	4	6	8	10	
0	1.00	0.92	0.78	0.58	0.32	0.01	0.602
30	0.99	0.90	0.74	0.54	0.29	0.00	0.577
50	0.99	0.88	0.71	0.50	0.26	0.00	0.557
70	0.98	0.83	0.61	0.40	0.20	0.00	0.503
80	0.97	0.74	0.48	0.27	0.12	0.00	0.430

(1983), removing apparent observer bias through geometric modeling procedures. The revised model presented in analytical form yields the distribution of relative frequency of CFLOS, $P_r(s, \theta)$, as a function of sky-cover, s, and zenith observing angle, θ, as follows:

$$P_r(s, \theta) = P_n^{(1 + b \tan \theta)}, \qquad (1)$$

where

$$P_n = 1 - s(1 + 3s)/4, \qquad (2)$$

and

$$b = 0.55 - s/2 . \qquad (3)$$

The resultant probability distribution of CFLOS is shown in Table 1.

For the objective at hand, this class of model approximation is a necessary first step, serving to convert climatic summaries of sky-cover into the corresponding climatic probabilities of ground to space CFLOS for designated viewing angles. The accuracy of the Lund-Shanklin and Allen-Malick models will be subjected to close analysis with the expanding data base generated by the WSI program, including the determination of systematic variations in the CFLOS relationships with respect to cloud type.

3.0 PROPERTIES OF THE ORNSTEIN-UHLENBECK MARKOV PROCESS

In this study, we explore a purely analytical solution to the problem of estimating the recurrence and persistence probability of sky-cover and CFLOS. The approach assumes that the stochastic behavior in both time and space can be described with close approximation by the Ornstein-Uhlenbeck Markov process (Feller, 1966). A brief review of the basic relationships as given by Gringorten (1972) is included here. The initial step is to transform the weather variable, (X), into its equivalent normal deviate (END), y, through its cumulative probability:

$$P_r(X \leq x) = \left(1/\sqrt{2\pi}\right) \int_{-\infty}^{y} \exp\left(-\eta^2/2\right) d\eta . \qquad (4)$$

The resulting new variable (y) has a variance of 1.0 and a mean of 0.0. In the O-U Markov process the correlation coefficient, ρ_t, between the two END's (y_o and y_t) separated by time interval, (t), is given by

$$\rho_t = \exp(-t/\tau) = \exp(-\alpha_t), \qquad (5)$$

where τ is the relaxation time.

The fundamental stochastic equation relating the END values of y_o and y_t over time interval (t) can be written (Gringorten, 1972)

$$y_t = \rho_t y_o + \sqrt{1-\rho_t^2}\ \eta_t , \qquad (6)$$

where η_t is the END of the conditional probability $P_r(y \leq y_t \mid y_o)$. The stochastic process is assumed to be stationary and the relaxation time is assumed constant. Eqs. 5 and 6 yield the conditional probability of a weather event following a prescribed initial condition. The input variables are the unconditional climatic cumulative frequencies of y_o and y_t, time interval (t) and a representative relaxation time (τ) for that location, month and time of day.

For continuous variables, such as temperatures, the value of y_o in Eq. 6 is uniquely defined by the cumulative probability distribution at time zero. For variables expressed in categories, such as sky-cover, it is important to subdivide the category probability range into subsets with smaller but equal probability ranges. The calculations of conditional probabilities should be carried out using in turn each of the midpoints of the subsets as y_o and the results averaged to yield the composite result for the sky-cover category. Experience shows that division into 6 subsets is sufficient for reliable results even for categories with a large range of unconditional probability.

In this study, Eq. 6 is used to determine the climatological probability of the recurrence of sky-cover and CFLOS in space as well as in time. Recurrence is defined here as the occurrence of a weather event at a later time or at another site following the occurrence of the event at initial time or at the reference location, without regard to conditions occurring over the intervening time or space. For joint occurrence estimates in space, the form of the expression is the same except that a relaxation distance is substituted for relaxation time.

The O-U Markov process is used also in this study to model the persistence of sky-cover and CFLOS. Persistence is defined as the uninterrupted presence of a weather condition at one site or a combination of sites. A mathematical solution of persistence probability for the O-U Markov process was presented by Keilson-Ross (1975). The solution for event duration probability is mathematically complex, such that Ross (1980) applied the method of cubic splines to approximate the solutions for more rapid calculation. Further analysis suggested to us an alternative analytical representation of the formal solution for reliable approximation over the desired range of output.

As given by the mathematical solution of Keilson-Ross, the case when the climatic cumulative frequency of the weather event is 50 percent ($y_o = 0.0$) is very simple:

$$F_\alpha (y_o = 0) = (1/\pi) \sin^{-1}[\exp -t/\tau)] , \quad (7)$$

where F_α is the unconditional probability that $y \leq y_o$ throughout time interval (t).

Solutions for $(y_o \neq 0)$ in this study are approximated by

$$f_\alpha (y_o) = f_\alpha (y_o = 0) + y_o (1 + 0.13 \alpha_\tau^{0.9}) ,$$

$$\begin{bmatrix} -2 \leq y_0 \leq 2 \\ 0 \leq \alpha_t \leq 3 \end{bmatrix} \quad (8)$$

where $f_\alpha (y_o)$ is the END value corresponding to $F_\alpha (y_o)$, and $f_\alpha (y_o = 0)$ is the END value corresponding to the solution for the maximum equal the median case, (y_o = 0), given by Eq. 7, and y_o is the END value corresponding to the cumulative probability of the weather event.

In turn, the conditional persistence probability of y remaining $\leq y_o$ in time interval t is given by

$$P_r (y \leq y_o , t \mid y \leq y_o) = F_\alpha (y_o) / P_r (y \leq y) \quad (9)$$

where $P_r (y \leq y_o)$ is the unconditional probability of $y \leq y_o$ at $t = 0$.

In contrast with the recurrence expression (Eq. 6), the persistence probability expression (Eq. 9) assumes that the initial and final climatic frequency distributions are the same. Work is underway to extend the analytic process to include the effects of systematic changes in the unconditional event probabilities.

4.0 ESTIMATING THE RECURRENCE AND PERSISTENCE FREQUENCY OF CFLOS FOR A GIVEN SKY-COVER CONDITION.

Let us direct attention to a unique subset of photogrammetically determined CFLOS data that has been summarized by Lund (1973). During one summer of the 3-year observational program at Columbia, MO, CFLOS determinations were made from whole-sky photographs at 5-min intervals between the hours of 0800 and 1700, for a total of 585 hours in June, July, August, and September of 1969. Recurrence and "5-min persistence" frequencies of CFLOS were calculated by Lund (1973) using a grid representing lines of sight at azimuth angles 0, 90, 180 and 270 deg over the zenith angle range 0-80 deg in increments of 10 deg. Cloud/no cloud determinations were made for each of the 33 grid points at 5-min time intervals. Data from all grid points were included in the summarized recurrence and persistence statistics without regard to grid point location.

Hourly observations by National Weather Service personnel provide concurrent determinations of sky-cover. For the 5-min data base, the relative frequencies of recurrence and persistence were calculated by Lund (1973) as a function of sky-cover. It was assumed that the sky-cover at all 5-min intervals during a given hour was the average of the conventional observations of sky-cover made at the beginning and the end of the hour.

Thus, the probability statistics summarized by Lund for the 5-min data do not reflect synoptic changes in sky-cover during the intervening time interval. The summaries do depict the relative frequencies of recurrence and persistence as observed at the grid points for fixed categories (tenths) of average sky-cover during the hourly periods.

4.1 Comparison of 5-min recurrence frequencies with O-U Markov model estimates.

The cloud-free and the cloud-obscured recurrence relative frequencies as a function of sky-cover extracted from Figs. 6 and 7 of Lund (1973) are given in Table 2. The observed frequency of CFLOS for this subset is 0.533. In general, the recurrence statistics appear to be quite regular and consistent except for some categories of sky-cover where the number of observations is very small.

The overall cloud free and cloudy recurrence probabilities for this summer season at Columbia, MO, denoted by "ALL" in Table 2, were calculated as the average weighted by the observed frequency of occurrence of the individual categories of sky-cover in tenths. The cloud free and cloudy recurrence probabilities are about the same for this data base where the event probabilities are roughly equal.

The relative recurrence frequencies as calculated by Eqs. 5 and 6 are shown in Table 3. A cloud-element relaxation time, τ (c) of 30 min was selected in a deliberate attempt to obtain a close fit for both cloud free and cloudy lines of sight. Our purpose here was to determine the extent the O-U Markov process effectively models the real recurrence behavior. The results shown in Table 3 compare well with the observed behavior given in Table 2 for the individual cloud categories as well as the overall recurrence probabilities.

Table 2. Cloud free and cloudy line of sight recurrence frequency as a function of sky cover at Columbia, MO. Data were extracted from Figs. 6 and 7 of Lund (1973). The period of record is 585 hours during the months of June, July, August and September 1969. The number of observations is denoted by N.

RECURRENCE PROBABILITY OF **CLOUD FREE** LINE OF SIGHT

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
5	100	98	93	92	85	82	79	72	71	71	67	89
15	100	95	91	87	81	76	72	60	61	60	55	84
25	100	95	90	86	79	74	67	56	54	54	46	82
35	100	96	90	86	78	70	64	55	50	50	45	81
45	100	97	87	85	76	69	60	55	45	45	45	80
55	100	97	85	85	73	71	60	59	40	40	41	79
N	26399	14288	16731	14256	10034	10283	11732	9456	6431	3041	909	123560

RECURRENCE PROBABILITY OF **CLOUDY** LINE OF SIGHT

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
5	64	64	61	68	61	63	74	83	88	94	98	87
15	46	60	52	52	50	50	66	77	85	90	97	82
25	44	53	47	47	45	43	58	73	82	89	97	79
35	28	50	44	43	44	38	55	67	80	90	96	77
45	10	55	35	38	42	34	47	61	77	89	95	74
55	9	61	29	39	43	33	50	58	75	90	95	74
N	133	1552	3069	3564	3826	4765	9256	14304	15745	12799	39087	108100

Table 3. O-U Markov model estimates of the relative recurrence frequency of cloud-free and cloudy lines of sight.

ESTIMATED RECURRENCE PROBABILITY OF **CLOUD FREE** LINE OF SIGHT (PERCENT)

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
	CFLOS PROBABILITY (PERCENT) 99.5	90	85	80	72	68	56	40	29	19	2.3	
5	100	96	94	92	90	88	84	78	74	68	49	91
15	100	94	90	88	83	81	74	64	57	49	24	84
25	100	92	88	85	80	77	68	57	48	39	14	81
35	100	92	87	83	77	74	65	52	42	33	9	79
45	100	91	86	82	76	72	62	48	38	28	7	77
55	99	91	86	82	75	71	60	46	36	26	5	77

ESTIMATED RECURRENCE PROBABILITY OF **CLOUDY** LINE OF SIGHT (PERCENT)

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
	CFLOS PROBABILITY (PERCENT) 0.5	10	16	20	28	32	44	60	71	81	98	
5	40	61	66	69	73	75	80	86	89	93	99	89
15	15	39	45	50	56	59	67	77	83	88	98	82
25	7	28	35	40	47	50	60	71	79	86	98	78
35	4	22	29	33	41	45	55	68	76	84	98	76
45	2	18	24	29	37	41	52	68	75	93	98	74
55	2	15	22	27	34	38	50	64	74	82	98	73

A variety of model deficiencies and data base anomalies lead some systematic disparities between model results and observed values. Attention is directed to one special consideration with respect to the CFLOS data base. Cloud/no cloud discrimination in the case of high thin clouds is at times very difficult for both the human observer and for determinations from whole-sky photographs. A particular case in point are instances when photogrammic results repeatedly specify cloud free conditions while the weather observer records obscuration by thin clouds or vice versa. This problem could contribute anomalously to the strong apparent recurrence of CFLOS in broken to overcast conditions as shown in Table 2.

4.2 Comparison of 5-min persistence values with O-U Markov model estimates.

We might expect that the O-U Markov model estimates of persistence probability for this data sample would verify equally well and would also confirm that the appropriate cloud-element relaxation time is near 30 min. We do have a special problem in that the cloud/no cloud determinations were made at 5-min intervals, such that we have no knowledge of conditions during the intervening time. Thus, for example, the so-called "5-min persistence" probability for the first 5-min interval is listed as being equal to the recurrence probability for that interval despite frequent undetected changes in cloudiness which contribute significantly to a relatively lower persistence probability for the interval.

A more representative comparison of model and observed persistence can be achieved by some adjustment to the "5-min persistence" data even though it is approximate and, therefore, introduces additional uncertainty in the comparison process. We can, for example, make an adjustment for the unknown fluctuations in CFLOS during the first 5-min period by multiplying the observed "5-min persistence" by a correction factor which also should be applied to values for all subsequent time periods. Adjustments for unknown CFLOS fluctuations in subsequent time intervals are more difficult and controversial, but on the other hand have less impact if left uncorrected. So for purposes of this comparison the observed "5-min persistence" frequencies were adjusted by a single correction factor given by the ratio of the persistence and recurrence probabilities as calculated by the O-U Markov model for the first 5-min interval. The correction factor thus determined is applied to the observed probabilities for all time intervals. The correction factor is independent in the sense that the relaxation time used for the adjustment was determined from the recurrence probability distribution as described

in Section 4.1 above.

The observed relative "5-min persistence" frequencies for cloud free and cloudy lines of sight as extracted from Figs. 4 and 5 of Lund (1973) and subsequently adjusted by the correction factor are shown in Table 4.

Table 4. Cloud-free and cloudy line of sight persistence probabilities as determined from 5-min interval data at Columbia, MO. Summarized data were extracted from Lund (1973) and adjusted for high frequency changes as discussed in the text. Period of record is the same as for Table 2.

ADJUSTED PERSISTENCE PROBABILITY OF **CLOUD FREE** LINE OF SIGHT (PERCENT)

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
CFLOS PROBABILITY (PERCENT)	99 5	90	85	80	72	68	56	40	29	19	2 3	
ADJUSTMENT FACTOR (PERCENT)	99 1	93 4	91 0	89 2	86 2	84 6	80 3	74 5	70 7	65 4	51.1	
5	99	92	85	82	73	69	63	54	50	46	34	78 5
15	98	88	76	71	59	51	46	33	33	32	21	68 2
25	98	84	71	63	47	38	35	22	25	24	16	61 6
35	98	83	66	58	43	29	28	17	20	23	14	57 7
45	97	80	61	53	35	21	24	13	15	20	12	53 6
55	97	79	57	46	32	12	20	12	13	18	11	50 9

ADJUSTED PERSISTENCE PROBABILITY OF **CLOUD OBSCURED** LINE OF SIGHT (PERCENT)

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
CFLOS PROBABILITY (PERCENT)	0 5	10	16	20	28	32	44	60	71	81	98	
ADJUSTMENT FACTOR (PERCENT)	53 3	58 7	63 2	65 8	69 6	71 3	76 2	81 9	85.6	89 4	97 5	
5	34	38	39	45	42	45	56	68	75	84	96	75.9
15	25	29	24	26	24	24	38	52	64	76	93	65.8
25	15	20	20	20	16	14	26	41	56	72	91	59 8
35	13	17	16	17	11	7	21	34	51	65	87	54 6
45	4	14	15	17	9	4	18	29	43	63	84	51 0
55	0	15	11	12	8	3	14	25	37	59	82	47 8

The data base is identical to that used for calculation of the relative recurrence probabilities. Values are listed for each sky-cover category in tenths and the composite result. The persistence probability for the combined data base was calculated as the average weighted with respect to the observed climatic frequency of sky-cover in tenths when a cloud free (cloudy) line of sight occurs at any grid point selected at random.

Shown in Table 5 for comparison with the Columbia data in Table 4, are the relative persistence probability distributions for cloud free and cloudy conditions determined from Eqs. 7 and 8. As in the case of the recurrence calculations, the relative frequency of CFLOS as a function of sky-cover and the climatic frequency of sky-cover were taken from the Lund summary of the summer subset of data (585 hours of observation). The relaxation

Table 5. O-U Markov model estimates of cloud free and cloudy line of sightperistence probabilities.

ESTIMATED PERSISTENCE PROBABILITY OF **CLOUD FREE** LINE OF SIGHT

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
5	99	90	86	82	77	74	68	58	52	45	26	79 9
15	98	80	72	67	60	56	46	34	27	20	8	66.1
25	97	72	62	56	47	43	33	22	16	11	3	57.2
35	95	64	54	47	38	34	24	14	10	6	1	50.3
45	94	58	46	40	31	27	18	10	6	3	1	45 2
55	93	52	40	33	25	21	13	6	4	2	0	40.8

ESTIMATED PERSISTENCE PROBABILITY OF **CLOUD OBSCURED** LINE OF SIGHT

TIME MIN	CLOUD COVER (TENTHS) 0	1	2	3	4	5	6	7	8	9	10	ALL
5	21	36	42	45	51	54	61	70	76	83	97	77 8
15	6	14	18	21	26	29	38	49	58	68	93	63.4
25	2	6	9	11	15	17	25	36	46	57	90	54.8
35	1	3	5	6	9	11	17	27	37	48	86	48.3
45	1	2	3	4	6	7	12	20	29	41	83	43 5
55	0	1	2	2	3	4	8	15	23	34	79	39 0

time (30 min) was assumed to correspond with cloud-element the determination made in the recurrence probability comparison.

Although there is good agreement between the model estimates and the adjusted observed values over the first few 5-min intervals, the observed persistence probability increases relative to the model estimates with increasing lag time. We might suspect that the uncertainties involved in the determination of pure persistence from data with a sampling interval of 5-min contributes to the underestimates of persistence probability. The correction factors are only a rough approximation and the factors deal only with high frequency fluctuations not likely to be detected during the first 5-min interval. The WSI data base with determinations of CFLOS at 1-min intervals will provide for a much closer examination of the problem.

Another factor that might contribute significantly to the apparent model under-estimates of persistence is the differences in the discrimination of CFLOS between the human observer and the photographic method, as discussed above. Repeated disagreement over several time intervals would lead to a false perception of event occurrence and duration.

The Keilson-Ross mathematical solution for the persistence probability assuming the O-U Markov process reveals that the nonlinear properties of the relationship are an important consideration in its application to spe-

cific problems. In particular, a calculation of persistence probability using the CFLOS frequency of the mean sky-cover as input will not yield a reasonable estimate of the composite result for all sky-cover categories. Compare, for example, the calculated probability distributions for a median cloud cover between 3 and 4 tenths for cloud free persistence and for a median of 8 tenths for cloudy persistence with the corresponding calculations for all categories combined in the last column labelled "all" of the summaries. The disparity is particularly large at longer time intervals when overcast (clear) and near overcast (clear) sky conditions make the overwhelming contribution to the continuation of the cloudy (clear) line of sight. Note for example in the lower Table 5, the 55-min persistence frequency of CLOS with the median initial sky cover of 8 tenths is 0.23 vs. 0.39 for the weighed average (or all) calculation.

The nonlinearities in the persistence model calculations are of interest for another reason. The data base is a summary of relative frequency over a grid of 33 points spread over azimuth angles up to 80 deg. As shown in Table 1, climatic frequency of CFLOS depends significantly on the zenith angle of observation. Model calculations of the composite without regard to zenith angle, as was done in this comparison study, will result in an underestimate of the overall persistence probabilities. In practice, the model calculations should be confined to specific points or small regions of the sky dome. The results can be combined later if required for a given application.

In view of all the above considerations, we cannot extract a clear measure of performance of Eqs. 7 and 8 for estimating persistence probability through comparison with the "5-min persistence" data base. This is particularly true since the attempt to adjust the observed data to compensate for the behavior between intervals helps to ensure compliance between calculated and observed values at short lag times. On the other hand, it is encouraging that the O-U Markov model handles the recurrence probability estimates very well, and there is no reason at this point to suspect that the Keilson-Ross mathematical solution of the process for persistence probability estimates will not work as well. Comparisons with CFLOS data obtained at more frequent intervals is required for definitive evaluation.

4.3 Comparison of 1-hour recurrence probabilities of CFLOS with O-U Markov model estimates

An examination of O-U Markov model recurrence estimates over longer time intervals, including the effects of the natural changes in sky-cover with time, can be made through comparisons with yet other summaries of the Columbia, MO, data base by Lund (1973). The hourly recurrence of CFLOS was determined from the same 33-point grid as in the 5-min summary. A much larger data base was used for this purpose, covering 3 years (885 days) of data from all seasons at the Columbia site. The climatic summary of sky-cover for this period is given in Table 6. Shown also in the same table is the probability of CFLOS for specific categories of sky-cover in tenths which were taken from the 5-min data sample (comparable data were not available for the 885-day sample).

Table 6. Summarized sky-cover frequencies extracted from Lund (1973). The data base includes 885 days in all seasons over a period of 3 years at Columbia, MO. The CFLOS probabilities as a function of sky cover were taken from the 5-min data base as listed in Table 4. All frequency values are given in percent.

SKY CVR TENTHS	CLIMATIC FREQUENCY SKY COVER	CUMULATIVE FREQUENCY SKY COVER	PROBABILITY CFLOS VS. SKY COVER
0	18.5	18.5	99.5
1	4.5	23.0	90.2
2	4.7	27.7	84.5
3	5.1	32.8	80.0
4	3.5	36.3	72.4
5	2.8	39.1	68.3
6	4.5	43.6	55.9
7	4.7	48.3	39.8
8	4.9	53.2	29.0
9	5.9	69.1	19.2
10	40.9	100.0	2.3

In the case of the hourly recurrence data, model estimates of CFLOS recurrence can be obtained readily by making some simplifying assumptions which do not significantly degrade the accuracy of the results. We note that the recurrence probability for the composite data base can be calculated using the relative climatic probability of CFLOS corresponding to the mean sky-cover as input with essentially the same result as proceeding with the calculations for individual sky-cover categories and combining the individual results.

Let us direct attention again to the calculations of CFLOS recurrence for a fixed sky-cover category in tenths (or the mean sky-cover) shown in Table 3. Note that the recurrence probability of CFLOS approaches the climatic frequency of CFLOS for that sky-cover category after a time interval greater than roughly 2 times the relaxation time. The cloud-element relaxation time for CFLOS for a fixed sky-cover condition is on the order of 30 min. Thus, we can assume reasonably that after a time

interval of 1 hour or so the observed recurrence probability will correspond rather closely with the climatic CFLOS probability for the prevailing mean sky-cover at that point in time.

Thus, the problem of estimating the CFLOS recurrence at 1-hour intervals is reduced to a determination of the mean sky-cover for the period following the observed occurrence of CFLOS at a point selected at random from the 33-point grid. The mean sky-cover after initial CFLOS occurrence will systematically change with time so as to approach the mean sky-cover for the complete sample as the time interval increases. Since the diurnal variation in the cumulative frequencies of sky-cover are not available for this data base, these effects are not dealt with in this comparison. The progression of median sky-cover with time is given by Eq. 6 with $\eta_t = 0$ so that

$$y_t(\eta_t = 0) = \rho_t y_o . \qquad (10)$$

where $y_t(\eta = 0)$ is the END of the cumulative probability of the median sky-cover at time t, following the occurrence of CFLOS at time = 0.

Using the available climatological information shown in Table 6, the median sky-cover at the time of an initial occurrence of CFLOS at a grid point is 0.282. The climatic cumulative probability that the sky-cover will be equal to or less than 0.282 tenths is 0.319, with a corresponding END value, y_o, of -0.47. The median sky-cover for the complete data base (885 days) is 0.616.

The calculations of the median sky-cover for each hour and the probability of CFLOS associated with the calculated median sky-cover are shown in comparison with the recurrence probabilities observed by Lund (1973) in Table 7. The sky-cover relaxation time, τ (s), of 16 hours chosen for the calculations produced a very close correspondence in the observed and calculated recurrence probability values. The basic considerations for the determination fo relaxation time for sky-cover, τ (s), and for CFLOS, τ (c), for a given sky-cover category are reviewed in a later section of this report.

Table 7. Calculated and observed values of CFLOS recurrence probability at 1-hr intervals. The observed recurrence probabilities were extracted from Table 1 of Lund (1973), and refer to the same data base as in Table 5 above. Y(t) is the cumulative probability of the calculated mean sky cover and y(t) is the corresponding END value determined from Eq. 1.

TIME HOURS	CALCULATED y (t)	Y (t)	CALCULATED MEDIAN SKY COVER	CFLOS RECURRENCE PROBABILITY	OBSERVED CFLOS RECURRENCE PROBABILITY
0	-0.47	0.319	28	100	100
1	-0.442	0.329	30	80	80
2	-0.415	0.339	33	78	78
3	-0.390	0.348	36	76	76
4	-0.366	0.357	38	74	74
5	-0.344	0.365	41	72	72
6	-0.323	0.373	41	70	71

5.0 ESTIMATING THE MULTISITE JOINT OCCURRENCE FREQUENCY OF SKY-COVER WITH A O-U MARKOV MODEL

The basic property of a simple Markov process is that the conditional probability of a state at any future time, given the present state, is not dependent upon any additional knowledge of behavior prior to initial time (cf. Kendall and Buckland, 1971). Extending the concept to space, the analogous assumption is that the conditional probability of a state at any site, given the state at the closest site, is not dependent upon knowledge of the state at more distant sites. Eq. 6 applies in exactly the same form except that "α_t" is redefined, α_d, as the ratio of site separation distance, d, and relaxation distance, D, (directly analogous to relaxation time τ). For the addition of a new site outside an existing network of sites, the separation distance, d, is defined as the distance between the new site and the perimeter of the area enclosing the existing network of sites. The perimeter is defined by straight lines connecting the outermost sites in the group.

5.1 Estimating the joint occurrence frequency of sky-cover at 2 sites

As part of a detailed study of the joint occurrence probabilities of weather events at multiple sites, Lund and Grantham (1980) published comprehensive summaries of the joint relative frequencies of sky-cover in central United States. In particular, the joint occurrence frequency of sky-cover ≥ 0.8 in winter was determined for a network of 7 stations from hourly observations made over a period of 13 years. The extensive data summaries provide a convenient and reliable basis to explore the applicability of the O-U Markov model for the determination of the joint occurrence frequency of sky-cover in space.

The relative climatic frequencies of winter sky-cover ≥ 0.8 for the individual sites as summarized by Lund and Grantham (1980) are listed in Table 8. Shown in Table 9 are comparisons of the observed 2-site joint frequencies with the estimates of joint probabilities determined with the O-U Markov model. Model estimates are

Table 8 . Relative frequency of winter sky cover ≥ 0.8 for selected locations in central United States. The summarized data were extracted from Lund and Grantham (1980).

Location		Frequency Sky Cover ≥ 0.8
EVV	Evansville, ID	0.598
BLV	Scott AFB, IL	0.564
STL	St. Louis, IL	0.566
COU	Columbia, MO	0.532
MKC	Kansas City, MO	0.507
TOP	Topeka, KS	0.502
DDC	Dodge City, KS	0.406

Table 9. A comparison of observed and model calculations of the joint occurrence frequency of sky cover ≥ 0.8 in winter at selected pairs of sites. The observed data were obtained from summaries of Lund and Grantham (1980). The model estimates were calculated with Eq. 6, assuming relaxation distances, D, of 500 and 650 miles.

Locations		Separation Distance Miles	Joint Occurrence Frequency		
			Observed	Estimated	
				D = 500 mi	D = 650 mi
BLV	STL	32	0.512	0.510	0.517
TOP	MKC	56	0.437	0.431	0.440
COU	STL	108	0.457	0.448	0.459
MKC	COU	120	0.419	0.412	0.424
BLV	EVV	130	0.479	0.474	0.486
COU	BLV	139	0.443	0.436	0.449
STL	EVV	162	0.469	0.462	0.476
TOP	COU	176	0.392	0.392	0.406
MKC	STL	228	0.395	0.386	0.407
DDC	TOP	251	0.299	0.304	0.319
MKC	BLV	259	0.387	0.384	0.400
COU	EVV	269	0.419	0.413	0.429
TOP	STL	283	0.374	0.379	0.395
DDC	MKC	305	0.284	0.292	0.308
TOP	BLV	313	0.368	0.372	0.388
MKC	EVV	389	0.373	0.374	0.391
DDC	COU	420	0.269	0.283	0.300
TOP	EVV	443	0.359	0.366	0.383
DDC	STL	524	0.268	0.282	0.298
DDC	BLV	552	0.264	0.279	0.296
DDC	EVV	678	0.264	0.280	0.296

shown for 2 assumed values of relaxation distance, illustrating the sensitivity of the estimates for a range of input values. In the case of 2-site joint occurrence, a designated relaxation distance of about 500 mi yields a very close approximation to the observed winter values. In other words, with the proper choice of relaxation distance the model works well in this case over the full range of separation distance from 32 to 678 miles.

5.2 Estimating the joint occurrence frequency of sky-cover at more than 2-sites.

Commensurate with the O-U Markov process and its application to conditional probability determinations in space, the joint occurrence probability of a state at a remote site, given the state of the nearest site is not dependent upon knowledge of the state at other more distant sites. To the extent that this is true, the joint probability of the occurrence of a weather event at more than 2 sites is simply the product of the probabilities of the individual station pairs. One necessary condition is that the separation distance for each new station added to an existing group of stations is the distance to the nearest existing station or the distance to the perimeter enclosing the existing group of stations, whichever is less. The determinations should proceed in such a way that each new station is outside the perimeter of the existing stations.

Given the relative joint occurrence probability for constituent pairs of stations, the frequency of simultaneous occurrence at all stations can be determined easily. The joint occurrence frequencies, P(i j), for 2 sites listed in Table 9 are given by

$$P(i\,j) = P(i)\,P(j|i) \tag{11}$$

where P (i) is the unconditional frequency at an individual station (see Table 8), and P (j | i) is the conditional probability that the event will occur at the second site given the occurrence of the event at the first site. The expression for joint occurrence frequency at 3 sites is

$$P(i\,j\,k) = P(i)\,P(j|i)\,P(k|j) \tag{12}$$

and so on.

The stations included in this comparison are virtually in a line configuration, so the station separation is always greater than the distance to the existing area perimeter. If this were not the case, the distance to the perimeter should be used as discussed above, and the unconditional event probability at the closest perimeter point should be determined by spatial interpolation of existing site data.

Thus, starting at one end of the station configuration and combining the pair probabilities as in Eq. 12, the resultant estimates of the joint occurrence frequencies for combinations of 4 stations are shown in Table 10 and for combinations of 6 stations in Table 11. Again, the observed occurrence frequencies were extracted from the summaries by Lund and Grantham (1980) for the winter data base covering a period of 13 years.

Table 10. Same as Table 9 except the joint occurrence frequencies are for various combinations of 4 sites.

Locations				Joint Occurrence Frequency Observed	Estimated D = 650 mi	Estimated D = 500 mi
BLV	COU	DDC	MKC	0.203	0.217	0.194
BLV	COU	DDC	TOP	0.202	0.218	0.195
BLV	COU	MKC	STL	0.344	0.334	0.313
BLV	COU	MKC	EVV	0.313	0.308	0.284
BLV	COU	STL	EVV	0.373	0.361	0.339
BLV	DDC	MKC	STL	0.207	0.226	0.200
BLV	DDC	MKC	EVV	0.188	0.209	0.186
BLV	DDC	STL	EVV	0.211	0.235	0.214
BLV	MKC	STL	TOP	0.323	0.323	0.296
BLV	MKC	TOP	EVV	0.296	0.299	0.274
COU	DDC	MKC	STL	0.210	0.222	0.200
COU	DDC	MKC	EVV	0.188	0.208	0.184
COU	DDC	STL	EVV	0.198	0.218	0.195
COU	MKC	STL	TOP	0.326	0.317	0.295
COU	MKC	TOP	EVV	0.297	0.297	0.272
DDC	MKC	STL	TOP	0.212	0.224	0.199
DDC	MKC	TOP	EVV	0.196	0.216	0.193
MKC	STL	TOP	EVV	0.295	0.297	0.268

Table 11. Same as Table 9 except the joint occurrence frequencies are for various combinations of 6 sites.

Locations						Joint Occurrence Frequency Observed	Estimated D = 650 mi	Estimated D = 500 mi
BLV	COU	DDC	MKC	STL	TOP	0.187	0.184	0.167
BLV	COU	DDC	MKC	STL	EVV	0.172	0.175	0.154
BLV	COU	DDC	MKC	TOP	EVV	0.169	0.170	0.134
BLV	COU	DDC	STL	TOP	EVV	0.171	0.175	0.154
BLV	COU	MKC	STL	TOP	EVV	0.271	0.250	0.238
BLV	DDC	MKC	STL	TOP	EVV	0.172	0.177	0.155
COU	DDC	MKC	STL	TOP	EVV	0.170	0.170	0.144
					average	0.187	0.186	0.164
					ratio	1.000	0.991	0.873

Very close agreement is found between the observed and estimated values for both 4-site and 6-site combinations. However, the best correspondence in both cases results from an assumed relaxation distance of about 650 miles rather than 500 miles as revealed by the 2-site model calculations. So the evidence in this example indicates that the conditional event probabilities for pairs of stations within the sample are not entirely independent. As might be expected, the joint occurrence frequency for a pair of sites increases to some extent if the event occurs jointly at a nearby pair of sites. The evidence here also suggests that an effective adjustment consists of an appropriate increase in relaxation time for multiple sites, which remains essentially the same regardless of the number of additional sites above 2.

6.0 TRIAL DETERMINATIONS OF JOINT OCCURRENCE STATISTICS FOR WSI SITES

In summary a series of trial calculations were made for selected WSI sites to illustrate the potential range of joint CFLOS occurrence statistics to be determined with the O-U Markov model. Holloman AFB, NM, Kirtland AFB, NM, and China Lake, CA, were chosen for analysis. The climatic frequency distributions of sky-cover for January and July at these stations are shown in Table 12. The values are monthly averages of hourly observations over a 10 year period. It is important to note that the

Table 12. Climatic cumulative frequency of sky cover for selected WSI sites in January and July.

Sky Cover Tenths	Holloman AFB, NM Jan	Holloman AFB, NM July	China Lake, CA Jan	China Lake, CA July	Kirtland AFB, NM Jan	Kirtland AFB, NM July
10	24.0	12.1	20.9	3.5	27.6	12.4
9	29.5	20.8	27.1	6.2	31.6	20.0
8	35.0	29.6	33.2	8.9	35.7	27.6
7	40.5	38.4	39.4	11.6	39.8	35.1
6	46.0	47.1	45.6	14.3	43.8	42.7
5	52.0	56.0	51.4	20.1	47.5	50.0
4	58.0	65.0	57.2	25.9	51.3	57.2
3	64.0	74.0	63.0	31.6	55.0	64.5
2	70.0	83.0	68.8	37.4	58.8	71.7
1	76.0	92.0	75.0	43.2	62.5	79.0
0	100.0	100.0	100.0	100.0	100.0	100.0
Mean Sky Cover (Tenths)	5.3	5.7	5.2	2.2	4.8	5.0

diurnal variations in average cloud cover were not considered in these trial calculations. These systematic variations of course do have significant impact on the persistence determinations and should be included for comprehensive analyses of these sites. While the diurnal range in mean sky cover is only about one tenth at the selected sites in January, the diurnal range increases to 2 to 4 tenths in July. In the absence of more complete information, the values for scattered and broken clouds were assumed to be equally divided among the individual categories of clouds in those ranges.

6.1 Persistence probability of cloudy line of sight for zenith viewing angle = 30 deg as a function of fixed and specified sky cover

The trial calculations were made for cloudy lines of site for an assumed zenith viewing angle of 30 deg, CLOS (30). For the determination of recurrence and persistence probability, the relaxation time for cloud elements, τ (c), and for sky cover, τ (s), must be specified. Calculations can be carried out separately for cloud element persistence and sky cover persistence using the nondimensional time scales of α_t (c) and α_t (s). The actual time interval can be determined later by multiplying by the appropriate relaxation times τ (c) and τ (s). To retain the convenient advantage of a nondimensional time scale in the composite calculations, we assumed that

$$\tau (s) = 32\,\tau (c) \qquad (13)$$

Thus for a sky-cover relaxation time of 16 hours, the corresponding value of cloud-element relaxation time is 30 min, and the value of α (s) for a time interval of 4 hours is 0.250. In general, this assumption is unduly restrictive and should not be used for accurate determinations. It was used here to explore conveniently the sensitivity of the calculations to variations in τ (s), which govern the frequency of long duration intervals.

The climatic probability of CLOS (30) as a function of sky cover (s) is given in the last row of Table 13. For example from Eqs. 1, 2 and 3, the probability of CLOS at zenith angle 30 deg. for a sky cover of 8 tenths is 71 percent.

Table 13. Persistence probability for cloudy line of sight at zenith angle = 30 deg as a function of sky cover and time. A detailed calculation is given in the text for the resultant value that is outlined by the rectangle.

TIME/τ (s)	SKY COVER (TENTHS) 9	8	7	6	5	4	3	2	1
0.005	85	76	69	62	56	50	44	37	30
0.016	72	$\boxed{58}$	47	39	32	25	20	14	10
0.031	58	41	30	22	16	11	7.2	4.5	2.6
0.063	39	21	12	7.2	4.2	2.3	1.3	0.7	0.1
0.125	18	6.0	2.2	0.9	0.4	0.2	0.1	---	---
0.250	4.2	0.5	0.1	---	---	---	---	---	---
0.375	0.4	0.2	0.1	---	---	---	---	---	---
Climatic Probability	85	71	58	46	36	26	17	10	4

The relative persistence probability of cloudy lines of sight at zenith angle = 30 deg, CLOS (30), as a function of fixed and specified sky cover and α_t (s) as determined from Eqs. 7, 8 and 9 are shown in Table 13. Extending the example in the previous paragraph, let the time interval (t) be 15 min and the cloud-element relaxation time, τ (c) be 30 min, then the related variables become:

$\alpha_t (c) = 0.5$		Eq. 5
$\rho_t (c) = 0.6065$		Eq. 5
$\tau(s) = 16$ hours		Eq. 13
$t/\tau (s) = 0.016$		
$s = 0.8$		given
$P_r (y \le y_o) = 0.71$		given
$y_o = 0.551$		prob. table (Eq. 1)
$F_\alpha (y_o = 0) = 0.2074$		Eq. 7
$f_\alpha (y_o = 0) = -0.816$		prob. tables (Eq. 1)
$f_\alpha (y_o) = -0.227$		Eq. 8
$F_\alpha (y_o) = 0.411$		
$P_r (y \le y_o, t \mid y \le y_o) = 0.579$		Eq. 9, corresponds to persistence probability in Table 13

6.2 Recurrence probability of cloudy line of sight for zenith angle = 30 deg.

The recurrence probabilities of CLOS (30) as a function of normalized time, α (s), were determined by Eqs. 5 and 6 using the procedures presented in Section 4.3. The conditional probability of sky-cover when CLOS (30) is observed at the WSI sites is given in Table 14. The resultant calculations of CLOS (30) recurrence probability for January and July as a function of α (s) are shown in Table 15.

Table 14. Relative frequency of sky cover when a cloudy line of sight is observed for a zenith viewing angle = 30 deg.

Sky Cover Tenths	Holloman AFB, NM Jan	July	China Lake, CA Jan	July	Kirtland AFB, NM Jan	July
10	54.7	28.0	49.4	2.0	66.3	32.0
9	10.6	17.2	12.4	14.4	8.3	16.6
8	8.9	14.3	10.4	12.1	6.9	13.9
7	7.3	11.8	8.5	9.9	5.7	11.4
6	5.8	9.4	6.8	7.9	4.5	9.1
5	4.9	7.4	4.8	12.9	3.2	6.6
4	3.2	5.4	3.5	9.4	2.3	4.8
3	2.1	3.6	2.4	6.2	1.5	3.2
2	1.2	2.1	1.4	3.6	0.9	1.9
1	0.5	0.9	0.6	1.5	0.4	0.8
0	0	0	0	0	0	0
Mean Sky Cover (Tenths)	8.3	7.3	8.59	5.5	8.8	7.5

Table 15. Recurrence probability of cloudy lines of sight at zenith angle = 30 deg. for selected WSI sites in January and July.

	Holloman AFB, NM		China Lake, CA		Kirtland AFB, NM	
TIME Δt (s)	Jan	July	Jan	July	Jan	July
0.005	91	86	92	79	94	88
0.016	85	78	88	66	89	80
0.031	81	71	84	55	86	73
0.062	77	64	80	46	83	68
0.125	72	60	75	37	79	62
0.250	66	58	69	30	69	58
0.375	63	56	66	26	64	55
0.562	59	53	62	21	58	52
0.750	55	50	58	19	54	48
1.500	48	47	48	16	44	42
Climatic Frequency	44	43	42	16	42	39

6.3 Joint probability of CLOS (30) for combinations of 2 and 3 sites

Trial calculations of the joint relative frequencies of sky cover and joint relative frequencies of CLOS (30) for pairs of WSI sites and a combination of the 3 selected sites are shown in Tables 16 and 17. The separation distances between WSI sites are as follows:

Holloman AFB - Kirtland AFB	155 mi
Holloman AFB - China Lake	691 mi
Kirtland AFB - China Lake	620 mi

A measure of the sensitivity of the calculations to the assumed value of sky-cover relaxation distance, D, is given by the individual determinations for D = 300 mi and D = 600 mi. This appears to be a reasonable annual range of expected values for this geographical area, with higher values in the winter and lower values in the summer months. The procedures for the joint occurrence frequency determinations are given in the Section 5.

Table 16. Joint relative frequencies of sky cover for individual pairs of WSI sites and for a network of 3 WSI sites. Values are shown for relaxation distances (D) of 300 mi and 600 mi.

	Overcast		Overcast And Broken		Overcast And Broken And Scattered	
Stations	Jan	July	Jan	July	Jan	July
Holloman-Kirtland						
D = 300 mi	14.6	5.0	30.1	30.0	55.3	76.3
D = 600 mi	16.6	6.8	34.0	33.9	58.3	77.7
Holloman-China Lake						
D = 300 mi	5.9	0.6	22.5	7.6	48.6	40.2
D = 600 mi	8.1	1.1	25.9	9.5	51.3	41.3
Kirtland-China Lake						
D = 300 mi	7.0	0.7	21.9	7.2	48.3	35.3
D = 600 mi	9.4	1.2	25.6	9.2	51.3	37.7
Holloman-Kirtland-China Lake						
D = 300 mi	3.7	0.3	15.1	5.1	42.8	34.1
D = 600 mi	5.7	0.6	19.9	7.3	47.8	37.1

Table 17. Same as Table 16 except for the joint relative frequencies of cloudy lines of sight at zenith angles = 30 deg.

Stations	January	July
Holloman-Kirtland		
D = 300 mi	22.7	18.3
D = 600 mi	26.4	20.9
Holloman-China Lake		
D = 300 mi	14.5	5.2
D = 600 mi	17.6	6.4
Kirtland-China Lake		
D = 300 mi	14.9	4.9
D = 600 mi	17.9	6.1
Holloman-Kirtland-China Lake		
D = 300 mi	7.9	2.2
D = 600 mi	10.6	3.1

The joint occurrence frequencies of CLOS (30) shown in Table 17 were determined directly from the joint occurrence frequency of sky-cover in Table 16 and the relative climatic frequency of CLOS (30) as a function of sky cover given in the bottom line of Table 13. The CLOS (30) frequency for clear sky is 0 and 1.0 for overcast sky. Although the sky-cover between sites is not independent, it is assumed (Lund, 1973b) that the arrangement of clouds over the sites is independent. For viewing paths with the zenith angle near 30 degrees or less, the assumption should be valid for site separations greater than about 20 miles. Thus, the joint occurrence frequency of CLOS (θ) for a given sky-cover category is given by the square of the climatic frequency for that cover and zenith angle.

6.4 Persistence probability of CLOS (30) at individual and multiple WSI sites

The procedure for the determination CLOS (30) persistence probabilities consists of 3 steps. First the probabilities that the sky-cover will persist in an amount equal to or above specific fractional values (in tenths) are determined with Eqs. 7 and 8. Sample calculations for Holloman AFB in January are shown in Table 18. Next, the values in Table 18 are weighted by the conditional probability that each sky cover is observed when CLOS (30) is observed (from Table 14). Then for each time

Table 18. Persistence probability of sky cover for Holloman AFB in January. See text for sample calculation of outlined value.

	SKY COVER (TENTHS)										
TIME/τ (s)	10	≥ 9	≥ 8	≥ 7	≥ 6	≥ 5	≥ 4	≥ 3	≥ 2	≥ 1	≥ 0
0.005	90	91	92	92	93	94	94	95	96	97	100
0.016	82	84	85	87	88	89	90	92	93	94	100
0.031	76	78	$\boxed{80}$	81	83	85	86	88	90	92	100
0.063	67	69	72	74	76	78	81	83	85	88	100
0.125	55	58	61	64	67	70	73	76	79	82	100
0.250	40	44	48	51	54	58	62	66	70	74	100
0.375	31	35	38	42	46	50	54	58	63	68	100
0.563	22	25	29	32	36	40	44	49	54	60	100
0.750	15	18	22	25	29	33	37	42	48	54	100
1.125	8.1	10	13	16	19	22	26	31	37	43	100
1.500	4.4	6.0	7.8	9.9	12	15	19	23	29	35	100
3.000	0.5	0.8	1.1	1.7	2.4	3.6	5.2	7.5	11	15	100
Climatic Cum.Prob.	24	30	35	40	46	52	58	64	70	76	100

interval, we obtain the product of the persistence probability of CLOS (30) for a given sky cover category (Table 13) and the expected relative frequency of sky-cover in that category which is obtained by subtraction of adjacent weighted cumulative values of sky cover for that time interval in Table 18. In turn the persistence probability of CLOS (30) for a given time interval is the sum of the above products over all sky-cover categories.

The resultant determinations of CLOS (30) persistence probabilities for individual WSI sites for January and July are given in Table 19. For illustration let us consider a set of sample calculations yielding a specific result (56 percent) in Table 19 for the persistence probability of CLOS (30) at Holloman AFB in January for a normalized time, α (s) = 0.031. The actual time would then be 30 min for τ (s) = 16 hours and τ (c) = 30 min (see Eq. 13). As a first step we must determine the persistence probability of <u>sky cover</u> for that season and location using Eqs. 7, 8 and 9 as given in Table 18. In particular, the input variables for the case where s $\geq$ 8 tenths are:

α_t (s)	= 0.031	given
s	$\geq$ 0.8	given
P_r (s $\geq$ 0.8)	= 0.35	Table 12
y_o	= - 0.385	Prob. tables
ρ_t	= 0.969	Eq. 5

and the result for this <u>conditional</u> probability that the sky cover will remain above 8 tenths at Holloman in January for 30 min (α_t = 0.031) as shown in Table 18 is

$$P_r(s \geq 0.8, t \mid s \geq 0.8) = 0.80. \quad \text{Eqs. 7, 8 and 9}$$

From Table 14, the relative frequency of s $\geq$0.8 when CLOS (30) is observed is

$$0.547 + 0.106 + 0.089 = .742, \quad \text{Table 14}$$

so that the relative probability that s $\geq$ 0.8 will be observed for the entire 30 min period following the occurrence of CLOS (30) at the onset of the period is

$$0.80 * 0.742 = 0.594.$$

The corresponding relative persistence probability for s $\geq$ 0.9 is

$$(0.547 + 0.106) * 0.78 = 0.509 \quad \text{Tables 14, 18}$$

and so forth for the other sky cover classes.

From Table 13, we find that the calculated recurrence probabilities of CLOS (30) for a fixed sky cover of 0.8 and t = 30 min, α_t (s) = 0.031, is 0.41. The comparable value for s = 0.9 is 0.58 and 1.0 for s = 1.0 (overcast). The overall persistence probability of CLOS (30) for the time interval α_t (s) = .031 as given in Table 19 for Holloman AFB in January is calculated as the weighted sum:

$$1.0 * 0.547 * 0.76 + 0.58\,(0.653 * 0.78 - 0.547 * 0.76)$$
$$+ 0.41\,(0.742 * 0.80 - 0.653 * 0.78) - - - - - - - - - - - -$$
$$- - - + 0.0\,(1.0 * 1.0 - .995 * .92) = 0.56$$

which corresponds to the number we set out to duplicate for this demonstration exercise.

Table 19. Persistence probability of a cloudy line of sight at zenith angle = 30 deg as a function of time. A detailed sample calculation is given in the text for the value outlined by the rectangle.

	Holloman AFB, NM		China Lake, CA		Kirtland AFB, NM	
TIME/τ (s)	Jan	July	Jan	July	Jan	July
0.005	80	73	79	65	83	74
0.016	67	56	65	45	72	57
0.031	$\boxed{56}$	41	53	30	62	43
0.062	43	27	40	18	51	29
0.125	32	16	28	9.5	39	18
0.250	22	9.0	19	4.8	29	10
0.375	17	6.2	14	3.0	22	7.1
0.562	12	3.8	9.7	1.7	16	4.5
0.750	8.4	2.5	6.7	1.0	12	2.9
1.125	4.5	1.1	3.4	0.4	6.3	1.3
1.500	2.4	0.6	1.8	0.2	3.6	0.6
3.000	0.3	0.0	0.2	0.0	0.4	0.1

The relative persistence probabilities for the joint occurrence of CLOS (30) at 2 or more sites are shown in Table 20. The determinations were made with the same

Table 20. Persistence probability of the joint occurrence of cloudy lines of sight at combinations of 2 and 3 sites. The assumed relaxation distance is 300 mi.

	Holloman AFB, NM And Kirtland AFB, NM		Kirtland AFB, NM And China Lake, CA		Holloman AFB, NM And Kirtland AFB, NM And China Lake, CA	
TIME/τ (s)	Jan	July	Jan	July	Jan	July
0.005	69	57	74	56	61	45
0.016	54	38	59	36	46	28
0.031	44	26	48	25	36	19
0.062	35	17	37	16	27	13
0.125	26	11	27	9.6	19	8.2
0.250	18	6.4	17	5.1	11	4.4
0.375	13	4.2	11	3.1	7.4	2.6
0.562	8.4	2.4	6.7	1.6	4.2	1.4
0.750	5.6	1.5	4.2	0.9	2.5	0.8
1.125	2.7	0.6	1.8	0.3	1.0	0.3
1.500	1.3	0.3	0.8	0.1	0.4	0.1
3.000	0.1	0.0	0.1	0.0	0.0	0.0

procedure except that the cumulative frequency distributions of initial sky-cover that are used for the persistence calculations are now the joint occurrence frequencies of sky-cover given in Table 16. Again, the arrangement of clouds above the sites but not sky-cover is assumed to be independent.

The results of the trial calculations of the joint occurrence persistence probability that are listed in Tables 19 and 20 are summarized in graphical form in Figure 1 for January and Figure 2 for July. The time scale for the graphs was determined by assuming a sky-cover relaxation time, τ (s), of 16 hours and a cloud-element relaxation time, τ (c), of 30 minutes. The calculated climatic frequency that cloudy lines of sight (θ = 30) will occur and persist as a function of duration interval are shown for a single site as well as for the joint occurrence of CLOS (30) at multiple sites. Notice for example in Fig. 1 for January that the estimated climatic frequency of a continuous joint occurrence of CFLOS (30) at Holloman AFB and Kirtland AFB reduces to 1 percent after about 13 hours, whereas the estimated frequency reduces to 1 percent after only 9 hours for the continuous joint occurrence at Kirtland AFB and China Lake, CA, and after 3-4 hours duration at the 3-site combination of Holloman, Kirtland and China Lake. For the July estimates in Fig. 2, the climatic frequency of continuous joint occurrence

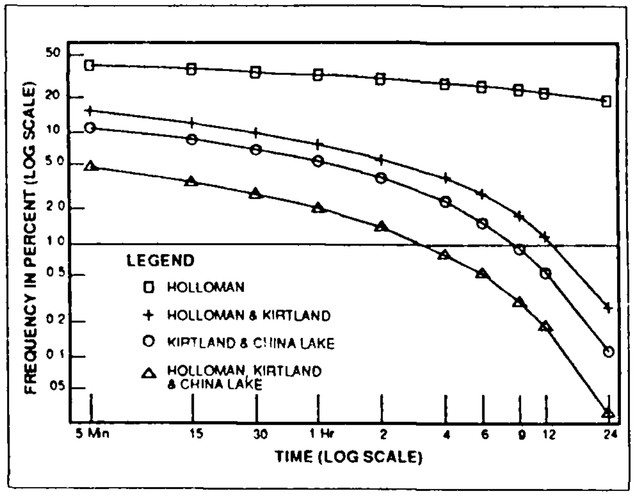

Fig. 1 Trial calculations of the probability of the continuous occurrence of CLOS (30) at single sites and the continuous joint occurrence at multiple sites as a function of duration interval. The determinations were based upon the climatic frequency of observed sky cover for January averaged over all hours at the individual sites.

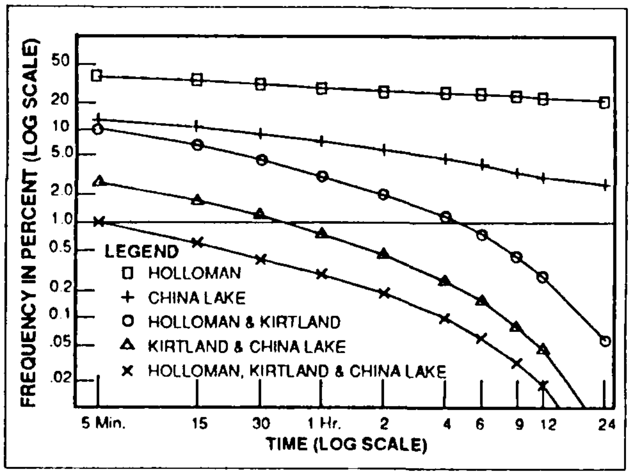

Fig. 2 Trial calculations of the probability of the continuous occurrence of CLOS (30) at single sites and the continuous joint occurrence at multiple sites as a function of duration interval. The determinations were based upon the climatic frequency of observed sky cover for July averaged over all hours at the individual sites.

of CLOS (30) reduces to 1 percent after a duration interval of about 4 hours at Holloman and Kirtland and after only 5-minutes duration at the 3-site combination of Holloman, Kirtland and China Lake.

6.5 Determination of down time duration frequency for individual or multiple sites

Estimates of downtime duration frequency can be obtained from Tables 19 and 20, where downtime is defined as the continuous occurrence of CLOS (30) at a site or at all sites in the case of multisite combinations.

The expected number of downtime episodes, N_x, of a given duration interval, I_x, in a given time period T (month, season, etc.) may be expressed

$$N_x = P_r(x) T \, P_r\left[CLOS(\theta)\right] / \left[\tau(s) \sum_{x=0}^{m} P_r(x) \bar{\alpha}_x(s) \right] \quad (14)$$

where m is the number of successive time intervals, x, and P_x (x) is the probability the event will persist for a period of time corresponding to x (difference between the persistence probability corresponding to beginning and ending time for I_x). P_r [CLOS (θ)] is the unconditional probability of a cloudy line of sight for zenith angle, θ, τ (s) is sky cover relaxation time and $\bar{\alpha}_s$ (s) is the antilog of the average of the logarithms of α_s (s) for the beginning and ending times for interval I_x (the beginning time for the first interval must be greater than 0; 1 - min is suggested).

The resultant trial determinations of downtime duration statistics for individual sites and multiple site combinations are summarized in Tables 21, 22 and 23. For purposes of illustration the input values to Eq. 14 for Holloman AFB in January (Table 21) for I_x = 15 - 30 min and τ (s) = 16 hours are as follows:

T = 1 - month or approximately 720 hours

P_r (x = 15 - 30) 0.67 - 0.56 = 0.11 (from Table 19)

P_r [CLOS (30)] = 0.44 (from Table 15)

$\bar{\alpha}_{15\text{-}30}$ (s) = 0.0221

Table 21. Estimated number of downtime episodes per month as a function of downtime duration interval for Holloman AFB, NM. Comparative estimates are given for assumed sky cover relaxation times of 16 and 20 hours. The relaxation distance is 300 miles.

HOLLOMAN AFB, NM					
TIME INTERVAL	HELAX TIME = 16 HR		TIME INTERVAL	RELAX TIME = 20 HR	
	January	July		January	July
1-5 MIN	19.7	57.2	0-6 MIN	15.7	45.8
5-15 MIN	13.2	38.8	6-19 MIN	10.5	31.0
15-30 MIN	11.1	31.2	19-38 MIN	8.9	25.0
30-60 MIN	12.1	31.1	38-75 MIN	9.7	24.9
1-2 HRS	11.1	23.8	1.2-2.5 HRS	8.9	19.0
2-4 HRS	9.6	14.7	2.5-5 HRS	7.7	11.8
4-6 HRS	5.3	6.1	5-7.5 HRS	4.2	4.9
6-9 HRS	5.1	5.0	7.5-11 HRS	4.1	4.0
9-12 HRS	3.3	2.9	11-15 HRS	2.7	2.3
12-18 HRS	3.9	3.0	15-22 HRS	3.1	2.4
18-24 HRS	2.0	1.3	22-30 HRS	1.6	1.0
24-28 HRS	2.1	1.1	30-60 HRS	1.7	0.9

Table 22. Same as Table 23 except for Kirtland AFB, NM, and China Lake, CA. The assumed relaxation time is 16 hours and the relaxation distance is 300 miles.

TIME INTERVAL	Kirtland AFB, NM		China Lake CA	
	January	July	January	July
1-5 MIN	12.2	45.7	22.8	48.0
5-15 MIN	8.5	30.8	15.9	27.3
15 30 MIN	7.3	24.9	13.3	19.4
30-60 MIN	8.3	25.1	14.2	17.1
1-2 HRS	8.3	19.8	12.6	11.4
2-4 HRS	8.0	12.9	10.2	6.4
4-6 HRS	4.7	5.6	5.4	2.4
6-9 HRS	4.7	4.7	5.0	1.8
9-12 HRS	3.2	2.7	3.2	0.9
12-18 HRS	3.8	2.8	3.6	0.8
18-24 HRS	2.0	1.2	1.8	0.3
24-28 HRS	2.3	1.1	1.8	0.2

Table 23. Estimated number of joint occurrences of downtime episodes per month as a function of downtime duration interval for combinations of 2 and 3 WSI sites. The assumed sky cover relaxation time is 16 hours and the assumed relaxation distance is 300 miles.

TIME INTERVAL	Holloman AFB, NM And Kirtland AFB, NM		Kirtland AFB, NM And China Lake, CA		Holloman AFB, NM And Kirtland AFB, NM And China Lake, CA	
	Jan	July	Jan	July	Jan	July
1-5 MIN	20.97	58.98	13.13	19.78	14.84	13.12
5-15 MIN	10.05	26.67	7.28	8.83	5.74	4.17
15-30 MIN	6.79	16.09	5.30	5.27	3.55	2.09
30-60 MIN	6.38	12.51	5.40	4.05	3.29	1.51
1-2 HRS	5.77	8.39	5.20	2.79	3.10	1.13
2-4 HRS	5.77	6.05	4.95	2.03	2.95	0.91
4-6 HRS	3.33	3.02	2.68	0.90	1.51	0.43
6-9 HRS	3.12	2.47	2.28	0.68	1.21	0.29
9-12 HRS	1.90	1.24	1.24	0.32	0.64	0.14
12-18 HRS	1.97	1.24	1.19	0.27	0.57	0.12
18-24 HRS	0.95	0.41	0.50	0.09	0.23	0.05
24-28 HRS	0.81	0.41	0.35	0.05	0.15	0.02

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surely not theirs. Indeed, the techniques presented in Sections 5 and 6 have not been subjected to critical review and remain to be tested extensively with data bases such as the new whole-sky imaging data base. Special thanks go to Irving Gringorten for corrections and additions to the basic equations given in Section 3 and for drawing attention to the formal mathematical solution by Keilson and Ross of persistence probability assuming the O-U Markov process. The author is also indebted to Carole Robb for expert assistance in the typing and formatting of the manuscript.

8.0 REFERENCES

Allen, J. H. and J. D. Mahlick, (1983). *The frequency of cloud-free viewing intervals*, Preprint 21st Aerospace Science Meeting, 10-13 Jan. 1983, Reno, Nev.

Boehm, Albert, Irving A. Gringorten and Charles F. Burger, (1986). Private communication.

Feller, W., (1966). *An introduction to probability theory and its applications*, Vol. II, John Wiley and Sons, Inc., New York, 626pp.

Gringorten, Irving I., (1966). *A stochastic model of the frequency and duration of weather events*, J. Appl. Meteorol. **5**:606-624.

Gringorten, Irving I., (1967). *Probabilities of moving time averages of a meteorological variate*, Tellus XX:461-472.

Gringorten, Irving I., (1968). *Estimating finite-time maxima and minima of a stationary Gaussian Ornstein-Uhlenbeck process by Monte Carlo Simulators*, J. Amer. Statistical Assoc., **63**:1517-1521.

Gringorten, Irving I., (1972). *Conditional probability for an exact noncategorized initial condition*, Monthly Weather Review, Vol. **100**, No. 1, 796-798.

Keilson, J., and H. F. Ross, (1975). *Passage time distributions for Gaussian Markov (Ornstein-Uhlenbeck) statistical process*, Selected Tables in Mathematical Statistics, Vol. **3**. Institute of Mathematical Statistics, American Mathematical Society, Providence, R. I.

Lund, I. A., 1973. *Persistence and recurrence probabilities of cloud-free and cloudy lines-of-sight through the atmosphere*, J. Appl. Meteor., 12, 1222-1228.

Lund, I. A., (1973b). *A model for estimating the joint probabilities of cloud-free lines-of-sight through the atmosphere*, J. Appl. Meteorol. 12:1040-1043.

Lund, I. A., and D. D. Grantham, (1980). *Estimating the joint probability of a weather event at more than two locations*, J. Appl. Meteor., 19, 1091-1100.

Lund, I. A., and M. D. Shanklin, (1973). *Universal methods for estimating probabilities of cloud-free-line-of-sight through the atmosphere*, J. of Appl. Meteor, **12**, 28-35.

Ross, H. F., (1980). *An algorithm for $F_\theta(y)$ using cubic B-Splines*, AFGL-TR-81-0068 AD A103468, Air Force Geophysics Laboratory, Hanscom AFB, MA.